

Curtin University, Semester 1, 2022
ECON 4002 (Dr. Lei Pan)
Problem Set 2
Due Friday, May 27th at 5:00pm AWST

Question 1. [30 marks] Ramsey model

Consider a model of a representative agent who maximises utility equal to $\sum_{t=0}^{\infty} \beta^t \log(c_t)$, where c_t represents consumption at t and $0 < \beta < 1$. There is a capital technology that produces ak_t^γ at t from k_t units of capital created at $t - 1$. The agent starts with a capital stock k_0 . The output from capital is taxed at the rate τ , with $0 < \tau < 1$. The tax revenue is used to finance wasteful government spending.

- (a) [10 marks] Write down the social planner's problem and find conditions that characterise the solution.
- (b) [15 marks] Verify that $k_{t+1} = sk_t^\gamma$ is a solution if s is a constant.
- (c) [5 marks] Find s .

Question 2. [40 marks] Ramsey model with endogenous labour

Consider a representative household who lives forever and is endowed with one unit of time every period which she spends on leisure, l_t , and work, h_t . This household derives utility from consumption and leisure which is given by:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t) + \gamma \log(l_t)]$$

where $0 < \beta < 1$ is the discount rate. There is a constant return to scale production technology that allows firms to produce goods given by $F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha}$ from K units of capital and H hours of labour. Firms must rent their capital and labour from households at the respective rental rates. The household owns all capital, there is full depreciation of capital and there is no labour augmenting neither productivity nor population growth.

- (a) [15 marks] Write down the representative household's budget constraint in a given period t , and formulate her utility maximisation problem. Find the equation that determines the consumption path.
- (b) [10 marks] Write down conditions that characterise firm's profit maximisation problem and market clearing conditions for the capital and labour market.
- (c) [15 marks] Derive the per capita capital accumulation for this economy. What is the steady state per capita capital and consumption in this economy?

Question 3. [30 marks] Lucas (1988) human capital accumulation model

A representative agent has 1 unit of time that he can allocate to produce consumption goods and accumulate education (human capital). The periodic utility function is given by $u(C_t) = \frac{C_t^{1-\theta}-1}{1-\theta}$, where $\theta > 1$ and the production function is given by $Y_t = K_t^\alpha (\phi_t H_t)^{1-\alpha}$ where H_t denotes human capital, ϕ_t is the fraction of hours devoted to work and $1-\phi_t$ is the fraction devoted to education. New human capital is produced using the CRS technology

$$H_{t+1} = B(1 - \phi_t)H_t$$

where $B > 0$ measures the return to education, and $H_0 > 0$ is given.

- (a) [5 marks] Write the aggregate resource constraint.
- (b) [25 marks] Solve the social planner's problem and find the optimal allocation of time between human capital and work. Show there is a balanced growth equilibrium where C_t , K_t , Y_t and H_t all grow at an identical rate.